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ANALYSIS OF VIBRATION ISOLATION SYSTEMS USING THE SENSITIVITY METHOD

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Abstract. *Evaluation of the sensitivity to parameter variations should be a basic aspect in the study of mechanical systems. In a complex system whose dynamical behavior is governed by several parameters it is important to know which ones are the most importants if we can change only a few parameters. In this work we study a passive vibration isolation system whose efficacy is measured by the transmitted power to the base of the system. The passive system is composed of two isolators and a rigid or flexible base, the excitation can have any direction. We use a substructuring technique to analyze the system. We have shown that the configuration (place of isolators, for example) and the type of excitation are very important in the design, and without the tool of sensitivity analysis would be very difficult to determine which parameters to modify. We have also studied the influence of some parameters, like mass, stiffness, isolators slope, etc.*

Keywords: *Vibration isolation, Vibratory power, Sensitivity analysis.*

1. INTRODUCTION

The sensitivity of a dynamic system related to its parameter variations should be a basic aspect in its analysis. The problem of sensitivity arises in the engineering field when mathematical models are used with analysis and synthesis purposes. With the aim to get a unique formulation of the problem, the mathematical model generally is assumed as exact, but it does not happen in fact because there is always a discrepancy between the model and the real system.

The mathematical model could be very poor if there is a considerable deviation between the model and the real system parameters, when the solution is very sensitive to this parameters. Therefore, it should be part of the modelling to know the parameters sensitivity beforehand to its implementation, or to reduce them in a systematic manner if it is necessary. This fact is still more important when it is done a optimization process because it maximizes or minimizes a performance index of a model for a specific set of parameters, so the solution will be strongly dependent of them.

In the analysis and comparison of the sensitivity, it is desirable to have a definition independent of the input signal form, in other words it must be defined in function of the structure of the system instead of input signals. This last condition is true for the definitions in the frequency domain. There are several definitions frequently used, and the most traditionally are (Frank, 1978): Bode, Horowitz and Perkins-Cruz sensitivity functions.

An interesting application of the sensitivity theory arises when it is necessary to modify a mechanical system in order to improve a given performance index (Naleckz & Wicher, 1988). For a complex system, it is difficult to evaluate the variation effect of each parameter, in this cases the sensitivity method appears as a valuable tool.

2. BASIC CONCEPTS AND DEFINITIONS

2.1. Mobility and impedance formulation

Considering "Fig. 2", it will be deduced the expressions used in this work a mobility (source and base) and impedance (isolators) matrix formulation (Bishop & Johnson, 1960).

All the forces and displacements have 3 components ($q_s = [f_y \;\; f_z \;\; t_x]^T$, $q_b = [f'_y \;\; f'_z \;\; t'_x]^T$, $f_{f1} = [n_{y1} \;\; n_{z1} \;\; t_{x1}]^T$, $v_{f1} = [v_1 \;\; w_1 \;\; \theta_{x1}]^T$, etc.), but, we have adopted a compact notation for simplicity. Because each component is a quantity that varies harmonically can be represented in the following way: $f_y = F_y e^{i\omega t}$, $f_z = F_z e^{i\omega t}$, $t_x = T_x e^{i\omega t}$, etc.

The expressions that relate the velocities ($\dot{v}$) and the forces (f) in the source and the base can be expressed in the following way (Gardonio et al., 1997; Coronado, 1999):

$$\dot{v}_s = M_{s1}.f_s + M_{s2}.q_s \tag{1}$$

$$\dot{v}_b = M_{b1}.f_b + M_{b2}.q_b \tag{2}$$

We have the following relations:

$$\dot{v}_s = \begin{bmatrix} \dot{v}_{s1} \\ \dot{v}_{s2} \end{bmatrix}, \quad f_s = \begin{bmatrix} f_{s1} \\ f_{s2} \end{bmatrix}, \quad \dot{v}_b = \begin{bmatrix} \dot{v}_{b1} \\ \dot{v}_{b2} \end{bmatrix}, \quad f_b = \begin{bmatrix} f_{b1} \\ f_{b2} \end{bmatrix} \tag{3}$$

The following notation is assumed: the mobility matrices (M) with subscript 1 correspond to internal forces and those with subscript 2 to external forces, whereas the velocities ($\dot{v}$) or the forces (f) with subscript 1 correspond to the left isolator and those with subscript 2 to the right isolator.

In the case of the isolators, the forces and the velocities in its ends will be related by the following way:

$$f_i = Z_i.\dot{v}_i \Rightarrow \dot{v}_i = Z_i^{-1}.f_i \tag{4}$$

Now, the general expressions that relates the external forces (q_s, q_b) with the internal velocities and forces $(\dot{v}_s, \dot{v}_b, f_s, f_b)$ will be deduced.

To simplify the notation we introduce the following matrices:

$$M_{sb1} = \begin{bmatrix} M_{s1} & 0 \\ 0 & M_{b1} \end{bmatrix}, \quad M_{sb2} = \begin{bmatrix} M_{s2} & 0 \\ 0 & M_{b2} \end{bmatrix} \tag{5}$$

$$q_{sb} = \begin{bmatrix} q_s \\ q_b \end{bmatrix}, \quad \dot{v}_{sb} = \begin{bmatrix} \dot{v}_s \\ \dot{v}_b \end{bmatrix}, \quad f_{sb} = \begin{bmatrix} f_s \\ f_b \end{bmatrix} \tag{6}$$

The first two equations can be grouped as:

$$\dot{v}_{sb} = M_{sb1}.f_{sb} + M_{sb2}.q_{sb} \tag{7}$$

Using the action and reaction principle for the forces and the continuity for the velocities, we get:

$$f_i = -T.f_{sb} \Rightarrow f_{sb} = -T^{-1}.f_i \tag{8}$$

$$\dot{v}_i = T.\dot{v}_{sb} \Rightarrow \dot{v}_{sb} = T^{-1}.\dot{v}_i \tag{9}$$

T is a transformation matrix that relates the displacements, velocities or forces of the masses, with those acting on the isolators extremities.

Using the above equations, one can find:

$$\dot{v}_{sb} = M_{\dot{v}}.q_{sb} \tag{10}$$

$$f_{sb} = M_f.q_{sb} \tag{11}$$

Where:

$$M_{\dot{v}} = (I + M_{sb1}.T^{-1}.Z_i.T)^{-1}M_{sb2} \tag{12}$$

$$M_f = -(T^{-1}.Z_i^{-1}.T + M_{sb1})^{-1}M_{sb2} \tag{13}$$

2.2. Power calculation

The net power in a period will be ("Fig. 1"):

$$P = \frac{1}{T}\int_0^T f_i\dot{v}_i\,dt = \frac{1}{T}\int_0^T Re[Fe^{i\omega t}]Re[\dot{V}e^{i\omega t}]\,dt \tag{14}$$

So, the power can be write as:

$$P = \frac{1}{2}Re[F^*\dot{V}] = \frac{1}{2}Re[F\dot{V}^*] \tag{15}$$

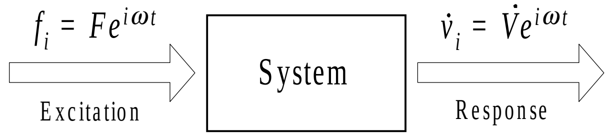

Figure 1: Power calculation

2.3. Bode sensitivity functions

In general, the matrix components of the sensitivity W_s are complex functions dependent on the frequency ω. This functions describe the influence of any parameter on the system dynamic properties characterized by a transfer matrix G.

Being $w_{s(k,l)}$ and $g_{k,l}$ the (k,l) components of the matrices W_s and G. Where $w_{s(k,l)} = \frac{\partial g_{k,l}}{\partial a_s}$, being a_s a parameter system.

The main idea is: if the magnitude of the sensitivity is high, then the change produced by the related parameter will be high too. In this case as the matrix G is the power transmitted to the base, all the components of the G and W_s matrices will be real.

The logarithmic sensitivity functions or Bode functions will be used in this work, they will be defined in this way:

$$S_{s(k,l)} = \frac{\partial ln(g_{k,l})}{\partial ln(a_s)} = \frac{\frac{\partial g_{k,l}}{g_{k,l}}}{\frac{\partial a_s}{a_s}} = w_{s(k,l)}\frac{a_s}{g_{k,l}} \tag{16}$$

3. ISOLATION MODELS

3.1. Model with a rigid base and two flexible isolators

The first system analyzed in this work considers the source and the base as rigid bodies under a multi-directional excitation (composed by a horizontal and a vertical forces, and by a moment) acting on the source. The source and base are joined by two flexible isolators, which have been modeled as a combination of a bar plus an Euler-Bernoulli beam.

In this system has been considered the following parameters to calculate the sensitivities: the source mass m_f, the source inertia moment Ixg_f, the isolators loss factor η_i, the isolators elasticity modulus Er_i, the isolators length L_i, the isolators area A_i, the base mass m_b, the base inertia moment Ixg_b, the base springs loss factor η_b, the base springs length L_b, the base springs elasticity modulus Er_b, the isolators location D_f and the isolators slope β.

The abscise of the figures will vary in a frequency range from 1 to 1000 Hz. The first three resonances (located near 10 Hz) are due to the source suspension, and the next two to the base suspension (over 100 Hz). Between this two regions it is located the isolation zone, where the excitation force should act.

It will be calculated the sensitivity of the power transmitted (the derivatives will be calculated numerically with respect to each parameter). The sensitivity will be showed in

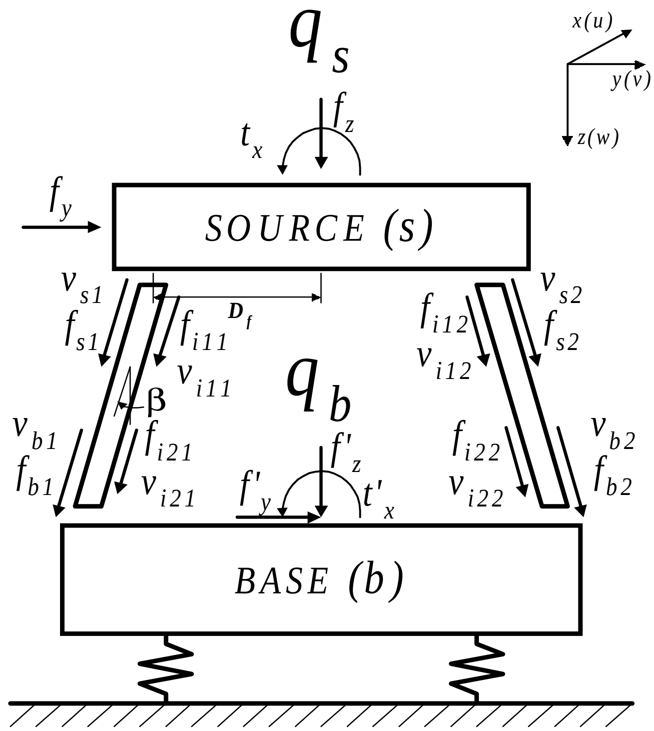

Figure 2: Isolation model with rigid base

a logarithmic scale and in absolute value (if they correspond to negative values we identify them by dashed lines).

The basic aim is to reduce the power transmitted to the base. Positive values mean that an increase on the parameter considered produces an increase in the power transmitted. Meanwhile negative values of the sensitivity mean that an increase on the parameter produces a diminution in the power transmitted.

Therefore, the procedure will be: identify the parameters with the biggest sensitivity (in value absolute), and then see the sensitivity sign, which will indicate whether this parameter must be increased or diminished.

Considering the multi-directional harmonic excitation $F_y = 1, F_z = 1, T_x = 1$ ("Fig. 3"), it can be noted that the source mass m_f has a great influence in the first two source suspension resonances, but afterwards it diminishes. The source inertia moment Ixg_f is important only near the third suspension resonance. The isolator loss factor η_i influence is smaller than the one of the other parameters, being relatively big and negative in the suspension resonances, afterwards positive and small in the isolation zone, and finally big in the isolator flexible resonances. The isolators elasticity modulus Er_i has rather sensitivity in the three suspension resonances, in the isolation zone, and in the isolator flexible resonances too, this sensitivity has a positive value, so it must be diminished, but this almost always has practical limitations because it produces a serious stability problem.

In "Fig. 4" it is seen the small and almost constant influence of the base springs loss factor η_b. The influence of the elasticity modulus of the base springs Er_b is important in the base resonances. The isolator location D_f shows appreciate influence in all considered frequencies with exception of those which are close to the base suspension resonances, moreover it is possible to see that this parameter is important near the first and third source suspension resonances. The isolators slope β show a great sensitivity near the three source suspension resonances, but it has a fall in the first base suspension resonance.

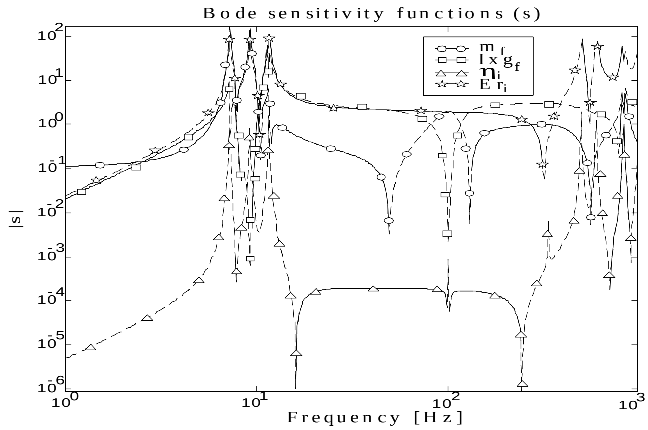

Figure 3: Sensitivity under excitation: $F_y = 1, F_z = 1, T_x = 1$

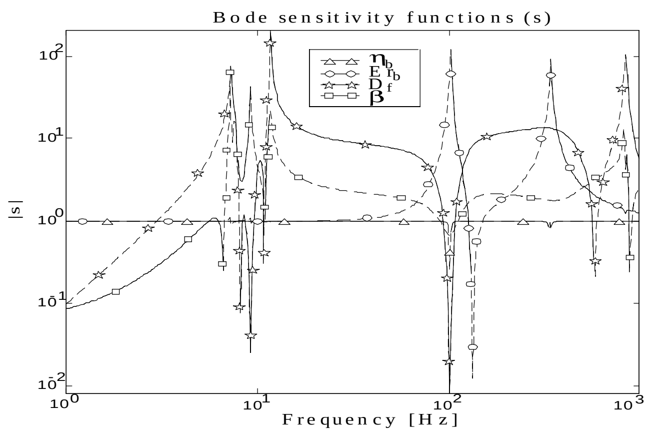

Figure 4: Sensitivity under excitation: $F_y = 1, F_z = 1, T_x = 1$

3.2. Model with a flexible base and two flexible isolators

A similar analysis as the one carried out for the system with rigid base will be done for a system with a flexible base ("Fig. 5"), since we have as objective to compare the parameters influence. The new base properties were chosen to obtain the first base resonance near to 100 Hz.

In "Fig. 6" it is shown that the source mass m_f has a similar influence as in the rigid model, excepting a fall in the 10-100 Hz frequency range. The source inertia moment Ixg_f, the isolator loss factor η_i and the elasticity modulus of the isolators Er_i show a similar behavior to first model too.

In "Fig. 7" it is showed that the transversal area of the base A_b is important only after the first base resonance located at 100 Hz. The elasticity modulus of the base Er_b is important in the second suspension resonance and in the flexible resonances. The isolator location D_f has a great sensitivity in the suspension resonances and in the isolation zone

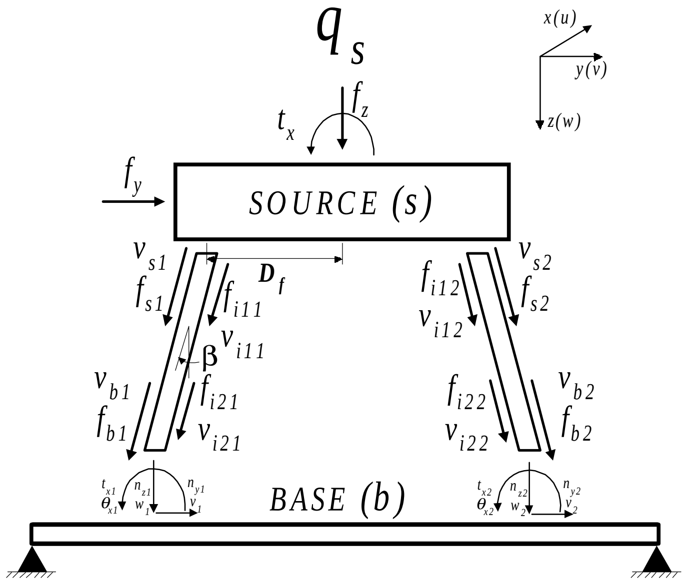

Figure 5: Isolation model with flexible base

(before 100 Hz). Finally, the influence of the isolator inclination angle β is similar to the previous parameter, but with smaller magnitude.

The isolation zone is generally between 1 and 100 Hz, and it is considered region of interest in projects of isolation systems (Inman, 1996). In this example, it will be calculated the sensitivity in this zone, therefore the first thing that will be made is to choose a frequency range where there is no big variation of the sensitivity, as it is the case far of the resonances.

In the following figures we have calculated 10 sensitivities in the 20-60 Hz isolation range, afterward it was calculated the mean of the absolute valor of them. It will be shown the influence of excitation force for the mean sensitivity.

In "Fig. 8" the excitation is harmonic and multi-directional $F_y = 1, F_z = 1, T_x = 1$, it is noted the great influence of the base length L_b get its maximum value at 100 Hz. The problem is that in many practical applications it is not possible to change this parameter due to external restrictions. Other parameters that show their importance in this region are: the isolators length L_i, the inertia moment of the source Ixg_f and elasticity modulus of the isolators Er_i. The isolator location D_f is important too, but not in the same level than when the base was rigid.

In the "Fig. 9" the excitation is a unitary force vertical $F_z = 1$, the base length L_b is again more important than the other parameters.

In "Fig. 10" when the excitation is a unitary moment $T_x = 1$, it is showed the importance of the isolator length L_i and of the base length L_b, here the isolator location D_f has a great sensitivity too.

Finally, four of the most important parameters were chosen, the base length was excluded because we supposed that it was not feasible to be modified. They are: Ixg_f, Er_i, L_i, D_f. The results are showed in the "Fig. 11", for the original system and for variations of 1 % and 5 % in each parameter.

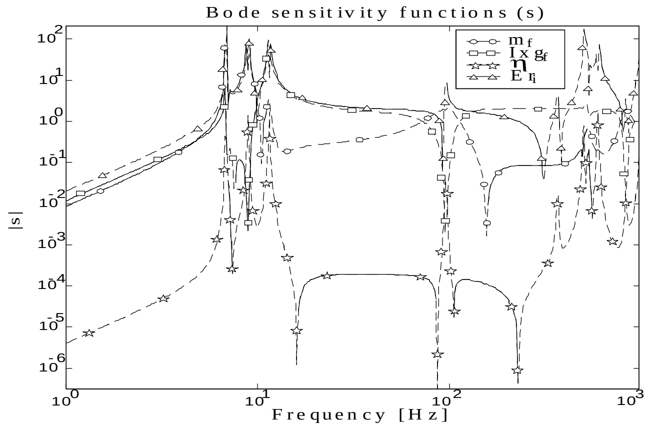

Figure 6: Sensitivity under excitation: $F_y = 1, F_z = 1, T_x = 1$

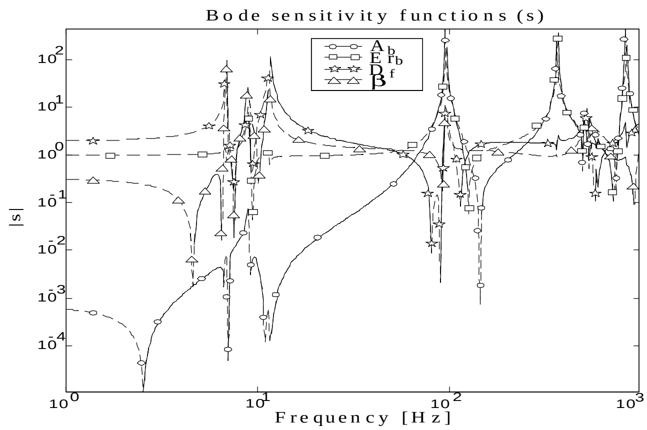

Figure 7: Sensitivity under excitation: $F_y = 1, F_z = 1, T_x = 1$

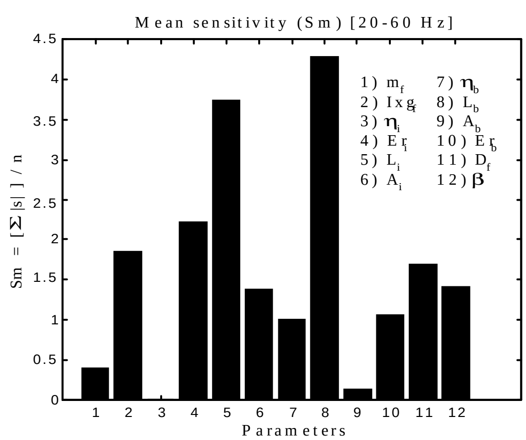

Figure 8: Mean sensitivity, under excitation: $F_y = 1, F_z = 1, T_x = 1$

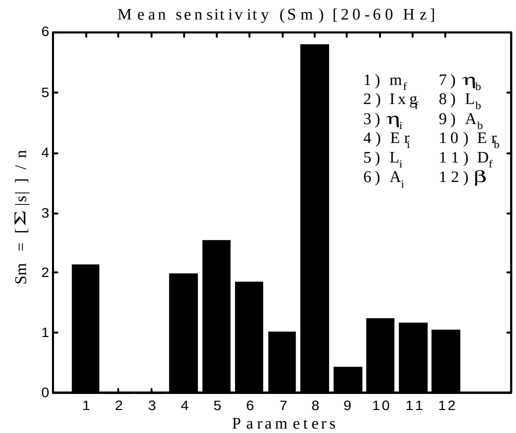

Figure 9: Mean sensitivity, under excitation: $F_y = 0, F_z = 1, T_x = 0$

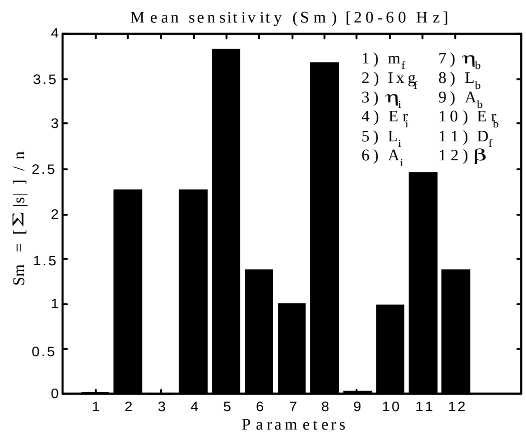

Figure 10: Mean sensitivity, under excitation: $F_y = 0, F_z = 0, T_x = 1$

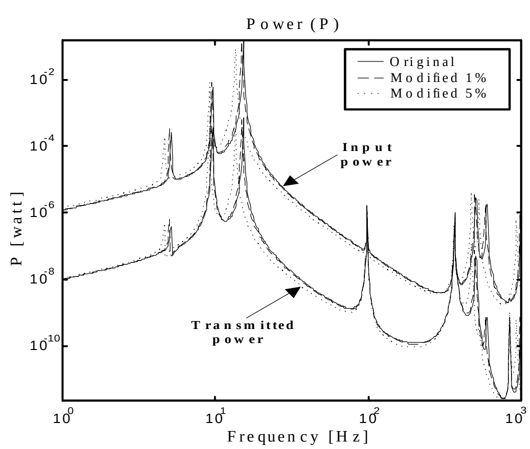

Figure 11: Comparison between the input and transmitted power, for some variations in the parameters: Ixg_f, Er_i, L_i, D_f, under excitation: $F_y = 1, F_z = 1, T_x = 1$

4. CONCLUSIONS

The sensitivity analysis was shown to be an important tool in the modification process of the parameters in the isolation systems analyzed in this work. It can be used for the choice of parameters that have the greatest influence either at a specific frequency or in a frequency range.

In general, it is not easy to choose the parameters that allow us to diminish the power transmitted without doing an analysis like this because the influence of each parameter depends strongly on the particular system configuration and on the excitation.

It was showed that for the same configuration a parameter could be important for a type of excitation and not for other one.

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